near the "Tour de Juvillac." A figure of the working-surface of the teeth of the lower jaw from this locality is given (of the natural size), showing the characters of the canine and proportions of the diastema. The close conformity in the characters of the upper grinders of the Puy-de-Dôme fossils of deposit with those of the Bruniquel cavern enables the author to dispense with figures of them.

The sum of the several comparisons is to refer the above Equine fossils from sedimentary deposits and both varieties from the Bruniquel cave to one and the same species or well-marked race belonging to the true Horses, or restricted genus *Equus* of modern mammalogists; the individuals of which race, with a small range of size, probably due to sex, were less than the average-sized horse of the present period, but larger than known existing striped or unstriped species of *Asinus*, Gray.

Interesting testimony, confirmatory of the conclusion from the palæontological comparisons, is adduced from outlines of the heads of different individuals of the Cave Equine when alive, neatly cut on the smooth surface of a rib of the same species, discovered by the Vicomte de Lastic St. Jal in 1863, in his cavern at Bruniquel, under circumstances which indisputably showed the work to have been done by one of the tribe of men inhabiting the cavern and slaying the wild horses of that locality and period for food.

The author remarks that every bone of the Horse's skeleton (and such evidence had been obtained from about a hundred individuals that had been exhumed at the period of his second visit to Bruniquel, in February 1864) had been split or fractured to gain access to the marrow. The dental canal and roots of the teeth had been similarly exposed in every specimen of jaw.

II. "On the Mechanical Possibility of the Descent of Glaciers, by their Weight only." By the Rev. Henry Moseley, M.A., Canon of Bristol, F.R.S., Instit. Imp. Sc. Paris, Corresp. Received October 21, 1868.

(Abstract.)

All the parts of a glacier do not descend with a common motion; it moves faster at its surface than deeper down, and at the centre of its surface than at its edges. It does not only come down bodily, but with different motions of its different parts; so that if a transverse section were made through it, the ice would be found to be moving differently at every point of that section.

This fact*, which appears first to have been made known by M. Rendu,

* The remains of the guides, lost in 1820 in Dr. Hamel's attempt to ascend Mont Blanc, were found imbedded in the ice of the Glacier des Bossons in 1863. "The men and their things were torn to pieces, and widely separated by many feet. All around them the ice was covered in every direction for twenty or thirty feet with the hair of one knapsack, spread over an area three or four hundred times greater than that of the knap-

Bishop of Annecy, has since been confirmed by the measurements of Agassiz, Forbes, and Tyndall. There is a constant displacement of the particles of the ice over one another, and alongside one another, to which is opposed that force of resistance which is known in mechanics as *shearing force.*

By the property of ice called regelation, when any surface of ice so sheared is brought into contact with another similar surface, it unites with it, so as to form of the two, one continuous mass. Thus a slow displacement of shearing, by which different similar surfaces were continually being brought into the presence and contact of one another, would exhibit all the phenomena of the motion of glacier ice.

Between this resistance to shearing and the force, whatever it may be, which tends to bring the glacier down, there must be a mechanical relation, so that if the shearing resistance were greater the force would be insufficient to cause the descent. The shearing force of cast iron, for instance, is so great that, although its weight is also very great, it is highly improbable a mass of cast iron would descend if it were made to fill the channel of the Mer de Glace, as the glacier does, because its weight would be found insufficient to overcome its resistance to shearing, and thus to supply the work necessary to those internal displacements, of which a glacier is the subject, or even to shear over the irregularities of the rocky channel. The same is probably true of any other metal.

I can find no discussion which has for its object to determine this mechanical relation between what is assumed to be the cause of the descent of a glacier, and the effect produced,—to show that the work of its weight (supposing that alone to cause it to descend) is equal to the works of the several resistances, internal and external, which are actually overcome in its descent. It is my object to establish such a relation.

The forces which oppose themselves to the descent of a glacier are, —1st, the resistance to the sliding motion of one part of a piece of solid ice on the surface of another, which is taking place continually throughout the mass of the glacier, by reason of the different velocities with which its different parts move. This kind of resistance will be called in this paper (for shortness) *shear*, the *unit* of shear being the pressure in lbs. necessary to overcome the resistance to shearing of one square inch, which may be presumed to be constant throughout the mass of the glacier.

2ndly. The friction of the superimposed laminæ of the glacier (which move with different velocities) on one another, which is greater in the lower ones than the upper.

3rdly. The resistance to abrasion, or shearing of the ice, at the bottom of the glacier, and on the sides of its channel, caused by the roughnesses

sack." "This," says Mr. Cowell, from whose paper read before the Alpine Club in April 1864 the above quotation is made, "is not an isolated example of the scattering that takes place in or on a glacier, for I myself saw on the Theodule Glacier the remains of the Syndic of Val Tournanche scattered over a space of several acres."

of the rock, the projections of which insert themselves into its mass, and into the cavities of which it moulds itself.

4thly. The *friction* of the ice in contact with the bottom and sides so sheared over or abraded.

If the whole mechanical *work* of these several resistances in a glacier could be determined, as it regards its descent, for any relatively small time, one day for instance, and also the *work* of its weight in favour of its descent during that day, then, by the principle of "virtual velocities" (supposing the glacier to descend by its weight only), the aggregate of the *work* of these resistances, opposed to its descent, would be equal to the work of its weight, in favour of it. It is, of course, impossible to represent this equality mathematically, in respect to a glacier having a variable direction and an irregular channel and slope; but in respect to an imaginary one, having a constant direction and a uniform channel and slope, it is possible.

Let such a glacier be imagined, of unlimited length, lying on an even slope, and having a uniform rectangular channel, to which it fits accurately, and which is of a uniform roughness sufficient to tear off the surface of the glacier as it advances. Such a glacier would descend with a uniform motion if it descended by its weight only, because the forces acting upon it would be uniformly distributed and constant forces*. The conditions of the descent of any one portion of it would therefore be the same as those of any other equal and similar portion. The portion, the conditions of whose descent it is sought in this paper to determine, is that which has descended through any given transverse section in a day; or, rather, it is one half this mass of ice, for the glacier is supposed to be divided by a vertical plane, passing through the central line of its surface, it being evident that the conditions of the descent of the two halves are the same. The measurements which have been made of the velocities of the surface-ice at different distances from the sides, make it probable that the differences of the spaces described in a given time would be nearly proportional to the distances from the edge in a uniform channel†; and the similar measurements made on the velocities at different depths on the sides that, under the same circumstances, the increments of velocity would be as the distances from the bottom. This law, which observation indicates as to the surface

* It is supposed that the weight is only just sufficient to cause the descent.

† Prof. Tyndall measured the velocity of the surface of the Mer de Glace at a series of points in the same straight line across it at a place called Les Ponts. The distances of these points in feet along the line up to the point of greatest velocity are set off to a scale in fig. 1; and the space in feet through which each point would pass in thirty-six days, if its velocity continued uniformly the same, is shown by a corresponding line at right angles to the other. The extremities of these last lines are joined. It will be seen that the line joining them is for some distance nearly straight; if it were exactly so, the law stated in the text would, in respect to this ice, be absolutely true. Fig. 2 shows in the same manner the spaces described in thirty-six days by points at different depths on the side of the Glacier du Géant, as measured by Prof. Tyndall at the Tacul. See Phil. Trans. Royal Society, vol. cxlix. part 1, pp. 265, 266. [The figures referred to in this note accompany the MS. of the paper.]

and the sides, is supposed to obtain throughout the mass of the glaciers. Any deviation from it, possible under the circumstances, will hereafter be shown to be such as would not sensibly affect the result.

The trapezoidal mass of ice thus passing through a transverse section in a day is conceived to be divided by an infinite number of equidistant vertical planes, parallel to the central line, or axis of the glacier, and also by an infinite number of other equidistant planes parallel to the bed of the glacier. It is thus cut into rectangular prisms or strips lying side by side and above one another. If any one of these strips be supposed to be prolonged through the whole length of the glacier, every part of it will be moving with the same velocity, and it will be continually shearing over two of the similar adjacent strips, and being sheared over by two others. The position of each of these elementary prisms in the transverse section of the glacier is determined by rectangular coordinates; and in terms of these, its length, included in the trapezoid. The work of its *weight*, while it passes through the transverse section into its actual position, is then determined, and the work of its *shear*, and the work of its *friction*. A double integration of each of the functions, thus representing the internal work in respect to a given elementary prism, determines the whole internal work of the trapezoid, in terms of the space traversed by the middle of the surface in one day, the spaces traversed by the upper and lower edges of the side, and a symbol representing the unit of *shear*. Well-known theorems serve to determine the *work* of the *shear* and the *friction* of the bottom and side in terms of the same quantities. All the terms of the equation above referred to are thus arrived at in terms of known quantities, except the unit of *shear*, which the *equation* thus determines. The comparison of this unit of *shear* (which is the greatest possible, in order that the glacier may descend by its weight alone) with the actual unit of *shear* of glacier ice (*determined by experiment*), shows that a glacier cannot descend by its weight only; its shearing force is too great. The true unit of shear being then substituted for its symbol in the equation of condition, the work of the force, which must come in aid of its weight to effect the descent of the glacier, is ascertained.

The imaginary case to which these computations apply, differs from that of an actual glacier in the following respects. The actual glacier is not straight, or of a uniform section and slope, and its channel is not of uniform roughness. In all these respects the resistance to the descent of the actual glacier is greater than to the supposed one. But this being the case, the resistance to shearing must be less, in order that the same force, viz. the weight, may be just sufficient to bring down the glacier in the one case, as it does in the other. The ice in the natural channel must shear more easily than that in the artificial channel, if both descend by their weight only; so that if we determine the unit of shear necessary to the descent of the glacier in the artificial channel, we know that the unit of

shear necessary to its descent by its weight only in the natural channel must be *less than that.*

A second possible difference between the case supposed and the actual case lies in this, that the velocities of the surface-ice at different distances from the edge, and at different heights from the bottom, are assumed to be proportional to those distances and heights; so that the mass of ice at any time passing through a transverse section may be bounded by plane surfaces, and have a trapezoidal form. This may not strictly be the case. All the measurements, however, show that if the surfaces be not plane, they are convex *downwards.* In so far therefore as the quantity of ice passing through a given section in a day is different from what it is supposed to be, it is greater than it. A greater resistance (other than shearing) is thus opposed to each day's descent, and also a greater weight of ice favours it; but the disproportion is so great between the work of the additional resistance to the descent, and that of the additional weight of ice in favour of it, that it is certain that any such convexity of the trapezoidal surface would necessitate a further reduction of the unit of shear, to make the weight of the actual glacier sufficient to cause it to descend.

A third difference between the actual glacier and the imaginary one, to the computation of whose unit of shear the following formulæ are applied, is this—that the formulæ suppose the daily motion of the surface of the glacier and the daily motion of its side to have been measured at the same place, whereas there exist no measurements of the surface motion and the side motion at the same place. The surface motion used has been that of the Mer de Glace at Les Ponts, and the side motion that of the Glacier du Géant at the Tacul—both from the measurements of Prof. Tyndall. This error again, however, tends to cause the unit of shear, deduced from the case of the artificial glacier, to be greater than that in the actual one; for the Glacier du Géant moves more slowly than the Mer de Glace. The quantity of ice which actually passes through a section at Les Ponts is therefore greater than it is assumed in the computation to be, whence it follows, as in the last case, that the computed unit of shear is greater than the actual unit of shear.

To determine the actual value of μ (the unit of shear in the case of ice) the following experiment was made. Two pieces of hard wood, each three inches thick and of the same breadth, but of which one was considerably longer than the other, were placed together, the surfaces of contact being carefully smoothed, and a cylindrical hole, $1\frac{1}{2}$ inch in diameter, was pierced through the two. The longer piece was then screwed down upon a frame which carried a pulley, over which a cord passed to the middle of the shorter piece, which rested on the longer. There were lateral guides to keep the shorter piece from deviating sideways when moved on the longer. The hole in the upper piece being brought so as accurately to coincide with that in the lower, small pieces of ice were

thrown in, a few at a time, and driven home by sharp blows of a mallet on a wooden cylinder. By this means a solid cylinder of ice was constructed, accurately fitting the hole. Weights were then suspended from the rope, passing over the pulley until the cylinder of ice was sheared across. As by the melting of the ice, during the experiment, the diameter of the cylinder was slightly diminished, it was carefully measured with a pair of callipers.

1st experiment.—Radius of cylinder ·65625 in., sheared with 98 lbs.

2nd experiment.—Radius of cylinder ·70312 in., sheared with 119 lbs.

By the first experiment the shear per square inch, or *unit* of shear, was 72·433 lbs.; by the second experiment it was 76·619 lbs. The main unit of shear of ice, from these two experiments, is therefore 75 lbs.

Now it appears by the preceding calculations, that to descend by its own weight, at the rate at which Prof. Tyndall observed the ice of the Mer de Glace to be descending at the Tacul, the unit of shearing force of the ice could not have been more than 1·3193 lb.*

To determine how *great* a force, in addition to its weight, would be necessary to cause the descent of a glacier of uniform section and slope, such as has been supposed in the calculations, let u represent, in inch-lbs., the *work* of that force in twenty-four hours. Then assuming the unit of shear (μ) in glacier ice to be 75 lbs., it follows, by the principle of virtual velocities, that

$$u = 94134000 + 1012560 - 2668400$$
$$= 92478160 \text{ inch-lbs.} = 7706513 \text{ foot-lbs.}\dagger$$

This computation has reference to half only of the width of the glacier, and to 23·25 inches of its length. The work, in excess of its weight, required to make a mile of the imaginary glacier, 466 yards broad and 140 feet deep, descend, as it actually does descend per twenty-four hours, is represented by the horse-power of an engine, which, working constantly day and night, would yield this work, or by

$$\frac{2 \times 7706513 \times 5280 \times 12}{23 \cdot 2 \times 24 \times 60 \times 33000} = 883 \cdot 78 \text{ h. p.}$$

The surface of the mass of ice, on which the work u is required to be done, in aid of its weight, to make it descend as it actually does, is 124771·5 square inches. The work required to be done on each square inch of surface, supposing it to be equally distributed over it, is therefore, in foot-lbs., $\frac{7706513}{124771 \cdot 5} = 61 \cdot 76.$

* By an experiment on the shearing of putty, similar to that which was made on the shearing of ice, its unit of shear was found to vary from 1 lb. to 3 lbs., according to its degree of hardness. If ice were of the same weight per unit of volume as soft putty, and its consistency about the same, it would descend by its weight *only* without the aid of any other force. It would not, however, be possible to walk on such ice.

† Thus the work to be done in aid of the weight is thirty-four times the work of the weight.

These 61·76 foot-lbs. of work are equivalent to ·0635 heat-units, or to the heat necessary to raise ·0635 lb. of water by one degree of Fahrenheit. This amount of heat passing into the mass of the glacier per square inch of surface per day, and reconverted into mechanical work *there*, would be sufficient, together with its weight, to bring the glacier down.

The following considerations may serve to disabuse some persons of the idea of an unlimited reservoir of force residing somewhere in the prolongation of a glacier backward, and in its higher slopes, from which reservoir the pressure is supposed to come which crushes the glacier over the obstacles in its way.

Let a strip of ice one square inch in section, and one mile in length, in the middle of the surface of the imaginary glacier, be conceived to be separated from the rest throughout its whole length, except for the space of one inch, so that throughout its whole length, except for that one inch, its descent is not retarded either by shear or by friction. Let, moreover, this inch be conceived to be at the very end of the glacier, so that there is no glacier beyond it. Now it may easily be calculated that this strip of ice, one inch square and one mile long, lying on a slope of $4° 52'$, without any resistance to its descent, except at its end, must press against its end, by reason of its weight, with a force of 194·42 lbs. But the cubical inch of solid ice at its extremity opposes, by the *shear* of its three surfaces, whose attachment to the adjacent ice is unbroken, a resistance of 3×75 lbs., or 225 lbs. That resistance stops therefore the descent of this strip of ice, one mile long, having no other resistance than this opposed to its descent, by reason of its detachment from the rest*. It is clear, then, that it could not have descended by its weight only when it *adhered* to the rest, and when its descent was opposed by the shear of its whole length; and the same may be proved of any number of miles of strip in *prolongation* of this. Also, with obvious modifications, it may be shown, in the same way, to be true of any *other* similar strip of ice in the glacier, whether on the surface or not, and therefore of the whole glacier.

It results from this investigation that the weight of a glacier is insufficient to account for its descent; that it is necessary to conceive, in addition to its weight, the operation of some other and much greater force, which must also be such as would produce those internal molecular displacements and those strains which are observed actually to take place in glacier ice, and must therefore be present to every part of the glacier as its weight is, but more than thirty-four times as great.

* If, however, the glacier were inclined at $35° 10'$, instead of $4° 52'$, and a strip were detached from its surface, as described above, it would equal the shear of one cubic inch at its lower end, if it were 300 yards long, and if the glacier were vertical, when it was 172·8 yards long.